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Research note

The Q-coverage multiple allocation hub covering problem with mandatory dispersion

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Backup coverage.

Abstract This paper addresses the multiple allocation hub set-covering problem considering backup coverage and mandatory dispersion of hubs. In the context of this paper, it has been assumed that a flow is covered if there are at least Q possible routes to satisfy its demand within a time bound. Moreover, there is a lower limit for the distance between hubs in order to provide a degree of dispersion in the solution. Mathematical formulation of this problem is given, which has $O(n^2)$ variables and constraints. Computational experiments carried out on the well-known CAB dataset give useful insights concerning model behavior and its sensitivity to parameters.

1. Introduction

Facility Location (FL) is a prime issue in a broad spectrum of production and service organizations, which traces its roots back to the beginning of the 20th century. It has a decisive role in the success of supply chains, with applications, including locating gas stations, schools, plants, landfills, police stations, etc. [1]. By nature, FL is regarded as a strategic issue. (A strategic or long-term decision does not take place on a regular basis and needs major capital investments [2].) A facility location problem encompassing social, political, and economic aspects can bring about great savings, and is not changed regularly. In fact, many tactical and operational decisions, such as vehicle routing and inventory control decisions, are directly influenced by decisions regarding the location of facilities. For a detailed introduction of location models, one may refer to valuable papers by Eiselt and Sandblom [3] or ReVelle and Eislet [4]. Moreover, ReVelle et al. [5] provide a comprehensive bibliography of recent publications in median, center and covering models, as three of the main location problem types. Another valuable review paper was recently published by Melo et al. [6] discussing FL models in supply chain management.

The Hub Location Problem (HLP) turned out to be a major area of interest for scholars after Goldman [7] in the late 60s. HLP has been a challenging topic in facility location. It arises in many different contexts, such as telecommunications, supply chain management, airline industry, computer networks, etc. A hub could be an airport, a seaport, a terminal, a warehouse, or any other facility. In a network of nodes sending and receiving flows, the most desirable option might be to allow direct exchange between any two points, but the cost of establishing such a complete network is unreasonably high [8]. HLP is an option to design networks, which is associated with finding locations for hub nodes and the allocation of non-hub nodes (spokes) to these hubs, so that the flow between origins and destinations becomes possible through hubs. In a traditional hub-and-spoke network, the direct flow between spokes is prohibited, and any flow must pass at least one of the established hubs. Such a strategy can lead to a reduced number of links, economies of scale, and more control of the network flow. In a hub-and-spoke network, there are economies of scale associated with using inter-hub links. This factor is represented as discount factor(s), which could have a tremendous impact on the solution of the problem, and is often shown as α. In a hub-and-spoke network, there are three components associated with the cost of each flow: collection cost (from the origin

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node to its allocated hub), transfer (hub node to hub node), and distribution cost (from hub node to the destination node).

A primary objective in siting service facilities is to ''cover'' as much of the potential customer demand as possible [9]. Traditionally, a covering type of problem looks for placement of facilities on a network or plane, so that all demands could be covered within a time limit. One relatively unstudied type of hub-and-spoke network is the hub-covering version. Normally, in a hub-covering problem, the coverage is achieved when one of the following criteria holds:

(i) The total cost/time between any origin–destination is below a value.
(ii) The cost/time for each link is limited.
(iii) The cost/time between any origin-hub and any hub-destination is less than a pre-determined value.

Some prominent applications of hub-covering are in the cargo sector, where flows must be traversed within a time bound, telecommunication networks, postal delivery services, less than truckload carriers, airlines, distributed computer networks, and reverse logistic networks. Hence, the problem of this paper could be employed by a variety of industries.

In a hub-and-spoke network, the allocation scheme is either single or multiple, based on the particular nature of the application. While, in single allocation networks, each spoke is allocated to one and only one hub node, in a multiple allocation scheme, sending and receiving flows is possible through more than a single hub. Although multiple-allocation is a popular and practical scheme, it has received relatively little attention so far. It is not hard to think of many applications, where the single-allocation scheme might be unrealistic. As an example, take traveling by air. Although passengers opt for a specific path to travel, they are free to choose their route among all possible paths in a network. In other words, there is often no restriction on the path to be selected. Moreover, due to many issues, such as airline traffic, a customer may be unable to select a specific route to take, in order to reach her/his destination on-time. Thus, many managers intend to provide alternative routes for their customers to travel within the time limits. The same issue holds in telecommunication networks where connecting origin–destination pairs using paths with bounded lengths is important for the quality of service [10]. In telecommunication networks, there is always a risk of failure in nodes, which necessitates the use of backup routes. Therefore, design of hub-and-spoke networks with backup coverage is inevitable in many real-world problems. In these kinds of network, there are alternative paths to be used, which can increase service levels in the network.

The potential advantage of the geographical dispersion of hubs is stressed by O'Kelly and Kim [11] who analyzed the recent cascading failure of Internet services in Korea in terms of network reliability [12]. They referred to the 2006 earthquake in Taiwan to show the significance of dispersion in the design of a hub-and-spoke network. In that earthquake, undersea cables were highly concentrated, and the earthquake caused severe telecommunications problems between continents. Another instance of a need for dispersion in hub-and-spoke networks is the design of emergency response warehouses in which dispersion is a prime issue to be considered. Hence, the dispersion among hubs can be a design consideration in many types of network.

A meticulous review of publications in the field of hub-and-spoke clearly shows that multiple-allocation schemes have received less interest to date. Furthermore, we have seen no papers in HLP literature discussing the concept of backup coverage. Additionally, to the best of our knowledge, the concept of mandatory dispersion among hub nodes has not gained much attention so far. Although various types of facility dispersion problems have been addressed in the literature, the rationale for the dispersion of hubs has rarely been explored [12]. Considering all these issues, the contributions of this paper could be defined as the definition of a novel hub location problem, the mathematical models of which are proposed, and analysis of the model performance regarding its parameters.

The outline of this paper is as follows: The paper proceeds with a concise literature review of HLP. Section 3 puts forward the definition of the problem and presents mathematical formulation. Numerical experiments and in-depth analyses of results appear in Section 4. Finally, conclusions and the outlook for potential future research are highlighted in Section 5.

2. Literature review

The Hub Location Problem (HLP) has been a focus area of researchers for more than forty years. From the beginning of the 21st century, an increase in the number of publications regarding the hub location problem has been witnessed. As already stated, the concept of HLP dates back to 1960s, when Goldman [7] defined the network hub location problem for the first time. Later, the study of HLP flourished by the seminal paper of O'Kelly [13] presenting the first mathematical formulation for a hub location problem. He studied the American airline passenger network and proposed the single allocation, p-hub, median problem, which has gained the most attention so far among various hub location problems. He also proposed a well-known dataset for hub location problems involving passenger flow between 25 US cities.

In this paper, we do not aim to review all the pertinent literature about network hub location problems and refer interested readers to valuable reviews by Campbell [14], Campbell et al. [15], Klincewicz [16], Alumur and Kara [17], etc. The continuous HLP was reviewed in papers by such as O'Kelly and Miller [18], Aykin and Brown [19], and Aykin [20]. The most recent review of network HLP is Alumur and Kara [17], which is an elegant and comprehensive review of more than 100 network hub location problems. They classified network hub location problems into four distinct sub-problems as p-hub median, hub location with fixed costs, p-hub center, and hub covering. For further details about the trend of HLP publications, interested readers are recommended to refer to [17] and references therein. In the last decade, some contributions have been made to hub location literature of which some influential publications are cited in Table 1.

As an example of using non-exact solution methods for HLP, Calik et al. [21] proposed a tabu-search based heuristic to solve the incomplete hub-covering location problem. Some other attempts to use heuristics and metaheuristics for HLP are: genetic algorithms by Cunha and Silva [22], ant colony by Meyer et al. [23] and variable neighborhood searches by Illic et al. [24]. The main thrust of these papers is to propose alternative solution methods for problems where exact solutions are handicapped to reach an optimal solution in a reasonable time. However, in some cases, more efficient formulations perform even better than non-exact solutions. An example is Alumur et al. [25] presenting a model for the same problem as Calik et al. [21]. The mathematical model of Alumur et al. [25] decreased the runtime needed considerably. Hence, finding

Table 1: Some fundamental extensions to the classical hub location problem over the recent decade.

Subject	Year	Author(s)
Latest arrival HLP	2001	Kara and Tansel [53]
Hub arc location problem	2005	Campbell et al. [54,55]
HLP considering congestion in hubs	2005	Elhedhli and Hu [56]
Latest arrival HLP with stopovers	2007	Yaman et al. [57]
Conditional p-hub median location problem	2009	Eiselt and Marianov [58]
Stochastic p-hub center with service level constraints	2009	Sim et al. [59]
Hierarchical p-hub median problem	2009	Yaman [10]
Reliable hub location problem	2009	Kim and O'Kelly [12]
HLP for time definite transportation	2009	Campbell [60]
Multiple allocation hub maximal covering problem	2009	Qu and Weng [61]
Efficient formulations of incomplete HLP	2009	Alumur et al. [25]
HLP with multiple capacity levels	2010	Correia et al. [62]
Competitive HLP in liner service providers	2010	Gelareh et al. [63]
Game theoretical model in HLP	2010	Lin and Lee [64]
Design of an intermodal hub-and-spoke network	2010	Ishfaq and Sox [65]
Stochastic uncapacitated HLP	2011	Contreras et al. [66]
Partitioning-hub-location-routing problem	2011	Catanzaro et al. [67]
Using fuzzy goal programming in reliable HLP	2011	Hansen and Mladenovic [68]
Ordered median HLP	2011	Puerto et al. [69]
$M/M/c$ queuing model for hub covering problem	2011	Mohammadi et al. [70]
p-hub median problem with integral constraints	2011	Lin et al. [71]
Hub network design of wagonload traffic	2011	Sender and Clausen [72]
Hierarchical HLP with fuzzy demands	2011	Davari and Fazel Zarandi [73]
Fuzzy dynamic virtual HLP	2011	Taghipourian et al. [74]
Hub covering location with different coverage types	2011	Karimi and Bashiri [75]
Single allocation hub location problem under congestion	2012	De Camargo and Miranda [76]
Uncapacitated multiple allocation p-hub median problem	2012	García et al. [77]

efficient exact models could still be vibrant in HLP literature compared to using non-exact solution methods.

While many publications have been dedicated to uncapacitated versions of HLP [26–32], there are others considering capacities for hub nodes or links, such as Costa et al. [33], Contreras et al. [34], Correia et al. [35], Randall [36], Yaman and Carello [37], and Yaman [38]. It is to be noted that our paper deals with an uncapacitated version of HLP.

Comparatively, the multiple-allocation scheme has been less considered in the literature. Similar to single allocation cases, Campbell [39] was the first to formulate the problem of a multiple allocation p-hub median problem. Later, Skorin-Kopov et al. [40] and Ernst and Krishnamoorthy [41] presented new models for the same problem. A special type of the problem was discussed in Sasaki et al. [42], considering a problem where each route in the network should use only one hub. Some other studies in multiple allocation cases are de Camargo et al. [32], Boland et al. [43], Marín et al. [44], and Cánovas et al. [45].

Multiple coverage models have been studied by many scholars, such as Kolen and Tamir [46], Gendreau et al. [47], Kim and Murray [48], Curtin et al. [49] and Erdemir et al. [50]. However, we have not seen any publication regarding backup coverage in HLP to date. In their recent valuable review paper, Zanjirani Farahani et al. [51] stated that multiple coverage models are very important areas of future research.

There have been some publications in literature regarding dispersion in siting facilities. Interested readers may refer to Curtin and Church [52] for a recent review of dispersion models in location problems. However, the only publication considering dispersion in HLP is by Kim and O'Kelly [12], which presents the p-hub reliable hub location problem with a mandatory dispersion model. Hence, there is a need to consider dispersion among facilities in locating hub nodes.

From the above, it becomes clear that the multiple-allocation HLP is still a relatively unexplored problem. Moreover, considering the review paper by Alumur and Kara [17] and our extensive review of the publications from 2008 to date, it becomes clear that backup coverage has not attracted the attention of researchers yet. Furthermore, the concept of dispersion is still rather untouched in the HLP literature. Hence, this paper considers these issues and proposes a novel formulation for a HLP, considering backup coverage and dispersion. Moreover, some analyses of the proposed model behavior are presented.

3. Problem definition

The objective of this paper is to locate hubs with the least cost where each demand is covered at least Q times. Take note that the application of this problem is when decision maker(s) aim to cover demands at least Q times, in order not to experience a disaster when emergencies occur, such as the failure of a hub to function or whenever there is a need to improve service levels, by providing more alternatives to satisfy a demand. Before giving the mathematical formulation of the problem, we define sets, parameters and variables as follows:

Sets

N: The set of demand nodes;
H: The set of potential hub nodes.

Parameters

β: Coverage radius;
M: A sufficiently big number;
d_{ij}: The distance to travel between nodes i and j;
Q: Number of times a node is to be covered;
d_{ij}^{km}: Distance needed to travel between nodes i and j using hubs k and m (k may equal m);
D_{Man}: The mandatory distance between hub nodes;
e_k: The fixed establishment cost of locating a hub at node k

$$a_{ij}^{km} = \begin{cases} 1 & \text{if } d_{ij}^{km} \le \beta \\ 0 & \text{otherwise.} \end{cases}$$

Variables

$$X_j = \begin{cases} 1 & \text{if a hub is located at } j \\ 0 & \text{otherwise.} \end{cases}$$

$$W_{km} = \begin{cases} 1 & \text{if both nodes } k \text{ and } m \\ & \text{ are selected to be hub nodes} \\ 0 & \text{otherwise.} \end{cases}$$

Now, the mathematical formulation of the problem is given as below:

$$\text{minimize } \sum_{k \in H} e_k X_k, \tag{1}$$

$$\text{Subject to: } \sum_{k \in H} \sum_{m \in H} a_{ij}^{km} W_{km} \geq Q \quad \forall i \in N, j \in N, \tag{2}$$

$$X_k + X_m \geq 2W_{km} \quad \forall k \in H, m \in H, k \neq m, \tag{3}$$

$$d_{km} + M(1 - X_k) + M(1 - X_m) \geq D_{\text{man}} \quad \forall k \in H, m \in H, k \neq m, \tag{4}$$

$$X_j \in \{0, 1\}, \tag{5}$$

$$W_{km} \in \{0, 1\}. \tag{6}$$

Objective Function (1) tries to minimize total cost of establishing hub facilities. Regarding the fact that the investment costs to establish facilities can be very high, finding solutions, in which the total cost is minimized, is of utmost importance. Since the cost of building links is trivial compared to the high cost of establishing hub nodes, the costs of building routes are not taken into account. Constraint (2), rules that each flow must be covered at least Q times. Clearly, the reliability of a network with backup coverage is higher than a network with a single coverage. Constraint (3) implies that the variable, W_{km}, takes a value of one if there are two hubs at nodes k and m. Constraint (4) governs that there should be at least a distance of D_{Man} between any two hub nodes. In other words, this constraint states that all solutions in which there are at least two nodes with a distance lower than D_{Man} should be removed from the feasible space. It is worth noting that this constraint is similar to the one in Kim and O'Kelly [12]. Constraints (5) and (6) are traditional constraints to limit X and W variables from taking binary values. It should be noted that the fixed establishment costs could be a function of the total flow entering each hub, or a fixed value. Assuming the worst case, where $H = N$, the model has $(n^2 + n)$ binary variables and $(3n^2 - 2n)$ constraints. Therefore, the model has $O(n^2)$ variables and constraints, overall.

4. Numerical examples

In this section, numerical examples are given and an analysis of problem parameters is provided.

4.1. Test problems

The well-known Civil Aeronautics Board (CAB) dataset is used to carry out experiments. The CAB dataset was first proposed by O'Kelly [78] and, since then, it has been consistently used in hub publications. It is based on airline passenger flow between 25 US cities in the 1970s. It is interesting to note that CAB is a subset of another dataset containing 100 cities in the United States. The selected 25 cities account for more than 50% of the total flow. Moreover, the distances between these cities satisfy triangle inequality and the flow is symmetric. Although there are other standard datasets, such as the Australian Post (AP) or Turkish network, this paper uses the CAB dataset to show the model behavior. Figure 1 shows the location of cities on the USA map which are considered in the CAB dataset.

In order to generate values for fixed establishment costs, it is assumed that the fixed cost to establish a facility is proportional to the amount of flow entering it. Thus, this parameter is generated using the normalized values of flow entering hubs between 0 and 10 000. It is to be stated that the values of β are similar to Kara and Tansel [79]. Moreover, each problem is solved for $Q = 1, 2, 3$ and $D_{\text{Man}} = 0, 250, 500, 750$ and 1000.

4.2. Computer specifications

All test problems were run on a 2 GHz Core2Duo CPU, equipped with 2 Gigabytes of RAM and using a CPLEX 12.1 solver.

4.3. Results, validation and discussions

In this section, numerical examples are given and results are analyzed in order to determine what impact, if any, problem parameters have on optimal locations and, also, runtimes. Tables 2–4 report the solutions obtained for various levels of problem parameters. Moreover, the runtimes of the problems are given separately in Table 5. Take note that in cases where there is no feasible solution for the problem, locations and runtimes are reported as N/A. Results are quite revealing, showing that Q and D_{Man} affect the solutions to a great extent.

An interesting analysis to be carried out is the effect of β on model output and the optimal solution. It appears from the yielded results that solutions are very sensitive to changes in the value of β. Results in Tables 2–4 testify that for lower values of β, there is a need to establish more facilities, which increases the fitness value. The same behavior could be observed for increasing the value of Q. Results bear out that being able to cover flows more than once, generally needs higher investment to establish more facilities. In order to show model behavior, the case with $\alpha = 0.2$ and $D_{\text{Man}} = 500$ is shown in Figure 2. Of special interest is the presence of some nodes, such as Seattle and Phoenix, in the hub set, for various pairs of (β, Q), which shows that results are somehow consistent for various levels of Q and β. This could be primarily attributed to the special scatter of nodes in the CAB dataset, which is sparser in the western part of the USA. For instance, since the distance between Seattle and other nodes in the CAB dataset are high, Seattle is a frequent location for hubs, regardless of the value for β. In other words, there are usually some hubs in western USA in order to exploit the advantage of economies of scale. This finding can be used by decision makers whenever there is a need to change the value of the coverage radius. To put it in simpler terms, considering the fact that some nodes are present among solutions, the cost of relocating hubs is less and there is no need to completely revise solutions. Further analysis showed that this behavior is general between various problems with different parameter settings. A remarkable finding is the fact that there is no solution for some instances of the problem, as reported in Tables 2–4. One instance are the problems with $\alpha = 0.8$, $\beta = 2307$ and $Q = 3$, with any value of D_{Man}.

At this point, we wish to turn to the study of the effect of mandatory distance on the solution. Looking at the results of Tables 2–4, it is apparent that the network turns out to be sparser in cases where D_{Man} is higher. As an instance, results of running the problem with $\alpha = 0.6$ and $\beta = 2557$ for various

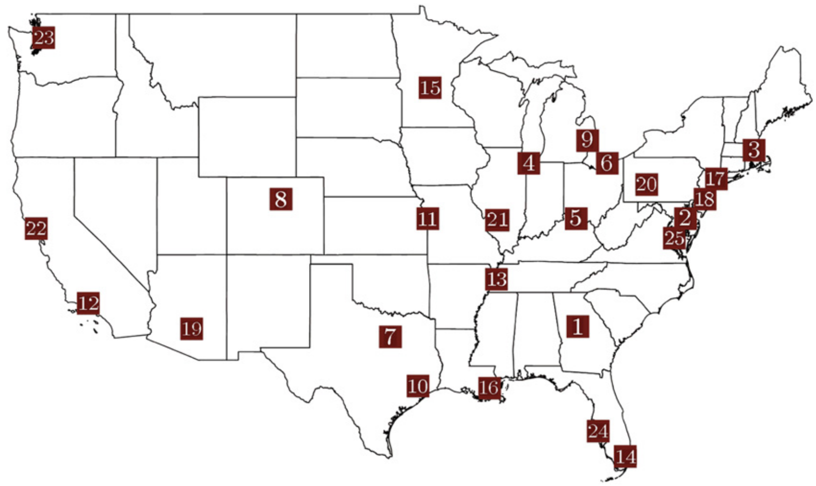

Figure 1: The CAB dataset on the USA map.

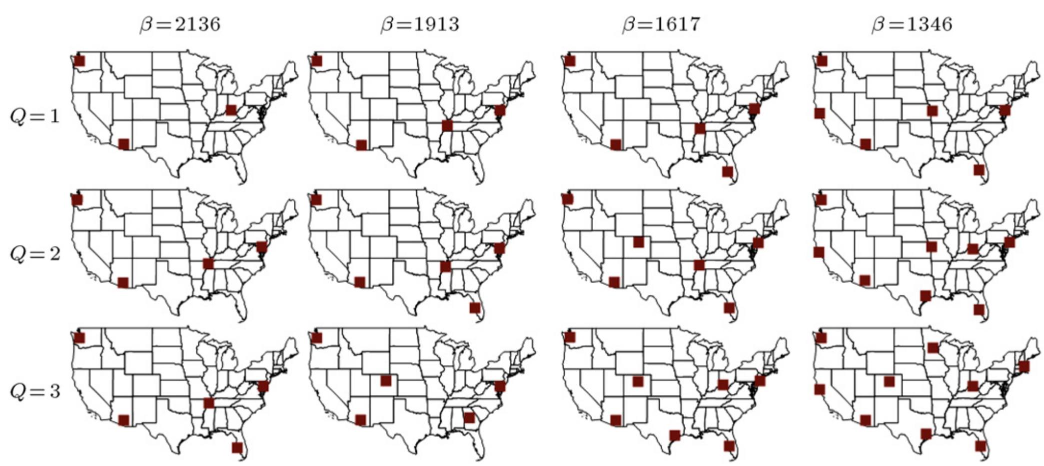

Figure 2: Comparing solutions with different values of β for the case with $\alpha = 0.2$ and $D_{\text{Man}} = 500$.

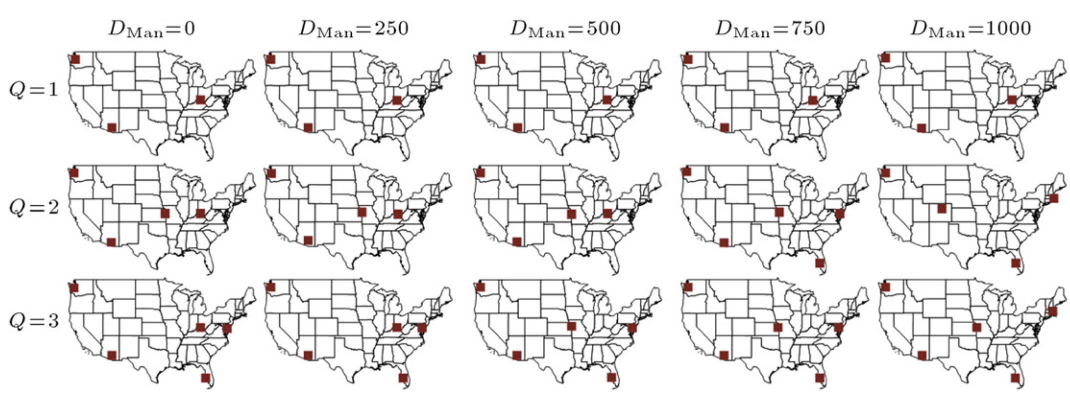

Figure 3: Comparing solutions with different values of D_{Man} for the case with $\beta = 2557$ and $\alpha = 0.6$.

pairs of (Q, D_{Man}) are depicted in Figure 3. A sample is for $Q = 3$, in Figure 3, where Baltimore is replaced by Boston, when D_{Man} is increased from 750 to 1000, so that the dispersion constraint is satisfied. Moreover, while in some cases, the number of hubs is increased, in cases when a larger D_{Man} is enforced on the model, in some others, the number of hubs is decreased, which seems somehow bizarre at first. An example is $Q = 2$ in Figure 3 by increasing the value of D_{Man} from 750 to 1000, where the number of hubs is decreased from five to four. It is to be noted that, as expected, although the number of hubs is less for $D_{\text{Man}} = 1000$, the fitness is higher. Another remarkable fact about the results is the increase in the number of hubs for higher values of Q, which could be easily justified considering the fact that normally, higher service levels need more hub nodes.

Another analysis is made about the total cost for different sets of parameters. Two sample analyses are plotted in Figures 4 and 5. Results clearly show that for a fixed choice of parameter pair (α, D_{Man}), the total cost of the network is increased when

Table 2: Optimal locations for $Q = 1$ and various levels of α, β and D_{Man}.

α	β	D_{Man}				
		0 Locations	250 Locations	500 Locations	750 Locations	1000 Locations
0.2	2136	5, 19, 23	5, 19, 23	5, 19, 23	5, 19, 23	5, 19, 23
0.2	1913	5, 13, 19, 23	5, 13, 19, 23	2, 13, 19, 23	2, 13, 19, 23	2, 10, 19, 23
0.2	1617	2, 13, 16, 19, 23	2, 13, 16, 19, 23	2, 13, 19, 23, 24	2, 11, 19, 23, 24	3, 11, 19, 23, 24
0.2	1346	2, 11, 19, 22, 23, 24	2, 11, 19, 22, 23, 24	2, 11, 19, 22, 23, 24	2, 11, 12, 23, 24	N/A
0.4	2401	13, 19, 23	13, 19, 23	13, 19, 23	13, 19, 23	13, 19, 23
0.4	2099	2, 8, 24	2, 8, 24	2, 8, 24	2, 8, 24	2, 10, 19, 23
0.4	1881	2, 13, 16, 19, 23	2, 13, 16, 19, 23	2, 13, 19, 23, 24	2, 11, 19, 23, 24	11, 14, 18, 19, 23
0.4	1597	2, 13, 19, 22, 23, 24	2, 13, 19, 22, 23, 24	2, 13, 19, 22, 23, 24	8, 12, 18, 21, 23, 24	N/A
0.6	2557	5, 19, 23	5, 19, 23	5, 19, 23	5, 19, 23	5, 19, 23
0.6	2336	2, 8, 24	2, 8, 24	2, 8, 24	2, 8, 24	2, 10, 19, 23
0.6	2184	8, 18, 23, 24	8, 18, 23, 24	8, 18, 23, 24	8, 18, 23, 24	8, 14, 18, 23
0.6	2002	2, 13, 19, 22, 23	2, 13, 19, 22, 23	2, 13, 19, 22, 23	N/A	N/A
0.8	2713	5, 19, 23	5, 19, 23	5, 19, 23	5, 19, 23	5, 19, 23
0.8	2552	2, 8, 13, 23	2, 8, 13, 23	2, 8, 13, 23	2, 8, 13, 23	8, 16, 18, 23
0.8	2457	5, 13, 19, 22, 23	5, 13, 19, 22, 23	2, 13, 19, 22, 23	3, 4, 8, 24	8, 17, 23, 24
0.8	2307	19, 20, 22, 23, 24	19, 20, 22, 23, 24	19, 20, 22, 23, 24	N/A	N/A
1	2826	11, 19	11, 19	11, 19	11, 19	11, 19
1	2762	11, 19	11, 19	11, 19	11, 19	11, 19
1	2726	8, 13, 23	8, 13, 23	8, 13, 23	8, 13, 23	5, 8, 23
1	2725	N/A	N/A	N/A	N/A	N/A

Table 3: Optimal locations for $Q = 2$ and various levels of α, β and D_{Man}.

α	β	D_{Man}				
		0 Locations	250 Locations	500 Locations	750 Locations	1000 Locations
0.2	2136	5, 13, 19, 23	5, 13, 19, 23	2, 13, 19, 23	2, 13, 19, 23	16, 18, 19, 23
0.2	1913	1, 5, 19, 23	1, 5, 19, 23	2, 13, 19, 23, 24	2, 11, 19, 23, 24	3, 11, 19, 23, 24
0.2	1617	2, 5, 11, 13, 19, 23, 24	2, 5, 11, 13, 19, 23, 24	8, 13, 18, 19, 23, 24	8, 12, 18, 21, 23, 24	N/A
0.2	1346	2, 11, 13, 19, 20, 22, 23, 24	2, 6, 11, 13, 19, 22, 23, 24	5, 10, 11, 18, 19, 22, 23, 24	N/A	N/A
0.4	2401	5, 13, 19, 23	5, 13, 19, 23	2, 13, 19, 23	2, 13, 19, 23	15, 19, 23, 24
0.4	2099	2, 13, 16, 19, 23	2, 13, 16, 19, 23	2, 13, 19, 23, 24	2, 11, 19, 23, 24	3, 11, 19, 23, 24
0.4	1881	8, 11, 18, 19, 23, 24	8, 11, 18, 19, 23, 24	8, 11, 18, 19, 23, 24	8, 12, 18, 21, 23, 24	N/A
0.4	1597	2, 11, 16, 19, 20, 22, 23, 24	11, 16, 18, 19, 20, 22, 23, 24	3, 6, 11, 14, 16, 19, 22, 23	N/A	N/A
0.6	2557	5, 11, 19, 23	5, 11, 19, 23	5, 11, 19, 23	2, 11, 19, 23, 24	3, 8, 23, 24
0.6	2336	2, 8, 19, 23, 24	2, 8, 19, 23, 24	2, 8, 19, 23, 24	2, 8, 12, 23, 24	11, 17, 19, 23, 24
0.6	2184	2, 5, 19, 22, 23, 24	2, 5, 19, 22, 23, 24	2, 16, 19, 21, 22, 23	8, 12, 18, 21, 23, 24	N/A
0.6	2002	2, 13, 16, 19, 20, 22, 23	2, 6, 13, 16, 19, 22, 23	3, 6, 13, 19, 22, 23, 24	N/A	N/A
0.8	2713	2, 5, 13, 19, 23	2, 5, 13, 19, 23	5, 8, 11, 23	2, 11, 19, 23, 24	3, 11, 19, 23, 24
0.8	2552	2, 8, 13, 19, 20, 23	2, 6, 8, 13, 19, 23	5, 11, 19, 22, 23	3, 8, 12, 21, 23, 24	N/A
0.8	2457	5, 11, 13, 19, 22, 23	5, 11, 13, 19, 22, 23	5, 11, 16, 19, 22, 23	N/A	N/A
0.8	2307	13, 14, 18, 19, 20, 22, 23, 24	N/A	N/A	N/A	N/A
1	2826	5, 8, 13, 19	5, 8, 13, 19	2, 8, 13, 19	2, 11, 19, 23	11, 18, 19, 23
1	2762	5, 8, 11, 19	5, 8, 11, 19	5, 8, 11, 19	11, 19, 20, 23	N/A
1	2726	8, 13, 14, 15, 23	8, 13, 14, 15, 23	8, 13, 14, 15, 23	6, 8, 10, 14, 23	N/A
1	2725	N/A	N/A	N/A	N/A	N/A

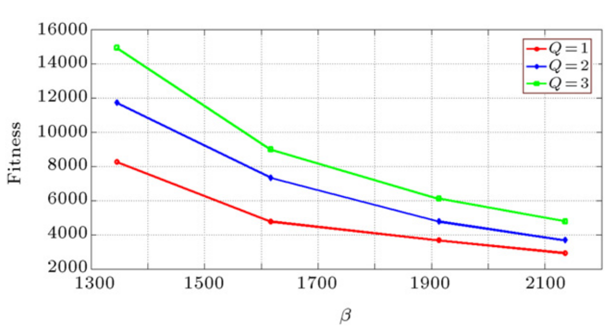

Figure 4: Comparing costs for $\alpha = 0.2$, $D_{Man} = 500$ for different levels of β and Q.

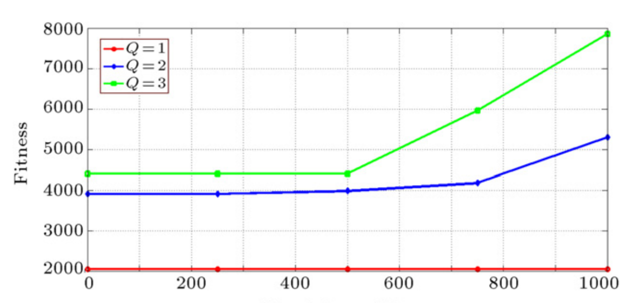

Figure 5: Comparing costs for $\alpha = 1$ and $\beta = 2826$ for different levels of D_{Man} and Q.

the value of β is decreased. This increase can be more than 200% in cases of Tables 2–4, which is a substantial change. This clearly shows that a decision to increase the service rate of this kind of hub-and-spoke network could be very high. In the same

Table 4: Optimal locations for $Q = 3$ and various levels of α, β and D_{Man}.

α	β	D_{Man}				
		0 Locations	250 Locations	500 Locations	750 Locations	1000 Locations
0.2	2136	2, 5, 13, 19, 23	2, 5, 13, 19, 23	2, 13, 19, 23, 24	2, 11, 19, 23, 24	3, 11, 19, 23, 24
0.2	1913	2, 5, 13, 16, 19, 23	2, 5, 13, 16, 19, 23	1, 2, 8, 19, 23	2, 11, 12, 23, 24	N/A
0.2	1617	2, 8, 13, 19, 20, 23, 24	2, 6, 8, 13, 19, 23, 24	5, 8, 10, 18, 19, 23, 24	3, 4, 8, 10, 12, 23, 24	N/A
0.2	1346	2, 5, 6, 8, 10, 19, 22, 23, 24	3, 5, 8, 11, 13, 19, 22, 23, 24	3, 5, 8, 10, 15, 19, 22, 23, 24	N/A	N/A
0.4	2401	2, 5, 13, 19, 23	2, 5, 13, 19, 23	2, 13, 19, 23, 24	2, 11, 19, 23, 24	3, 11, 19, 23, 24
0.4	2099	2, 5, 13, 19, 23, 24	2, 5, 13, 19, 23, 24	2, 8, 13, 19, 23, 24	10, 12, 15, 18, 23, 24	N/A
0.4	1881	1, 2, 5, 13, 19, 22, 23	1, 2, 5, 13, 19, 22, 23	1, 2, 8, 10, 11, 19, 22, 23	N/A	N/A
0.4	1597	1, 2, 11, 13, 16, 18, 19, 20, 22, 23, 24	1, 2, 3, 5, 11, 16, 19, 22, 23, 24	N/A	N/A	N/A
0.6	2557	2, 5, 19, 23, 24	2, 5, 19, 23, 24	2, 11, 19, 23, 24	2, 11, 19, 23, 24	3, 11, 19, 23, 24
0.6	2336	5, 11, 13, 19, 22, 23	5, 11, 13, 19, 22, 23	8, 13, 19, 20, 22, 23	3, 8, 12, 13, 14, 23	N/A
0.6	2184	2, 5, 19, 21, 22, 23, 24	2, 5, 19, 21, 22, 23, 24	5, 11, 18, 19, 22, 23, 24	N/A	N/A
0.6	2002	2, 6, 13, 16, 19, 20, 22, 23, 24	2, 3, 5, 13, 16, 19, 22, 23, 24	N/A	N/A	N/A
0.8	2713	2, 8, 11, 13, 19	2, 8, 11, 13, 19	5, 8, 11, 19, 23	8, 18, 21, 23, 24	N/A
0.8	2552	5, 8, 11, 13, 22, 23	5, 8, 11, 13, 22, 23	5, 8, 11, 16, 22, 23	N/A	N/A
0.8	2457	5, 11, 13, 19, 21, 22, 23	2, 5, 11, 13, 16, 19, 22, 23	5, 8, 11, 18, 19, 22, 23, 24	N/A	N/A
0.8	2307	N/A	N/A	N/A	N/A	N/A
1	2826	5, 8, 11, 19	5, 8, 11, 19	5, 8, 11, 19	11, 19, 20, 23, 24	3, 11, 19, 23, 24
1	2762	5, 8, 11, 23	5, 8, 11, 23	5, 8, 11, 23	8, 10, 20, 23, 24	N/A
1	2726	8, 9, 11, 14, 23	8, 9, 11, 14, 23	8, 9, 11, 14, 23	3, 8, 12, 14, 21, 23	N/A
1	2725	N/A	N/A	N/A	N/A	N/A

Table 5: The runtimes of the problems.

α	β	$Q = 1$					$Q = 2$					$Q = 3$				
		D_{Man}					D_{Man}					D_{Man}				
		0	250	500	750	1000	0	250	500	750	1000	0	250	500	750	1000
0.2	2136	51.698	44.538	28.455	15.132	6.069	84.131	61.761	49.375	13.136	1.95	67.517	69.311	42.963	15.351	1.482
0.2	1913	49.702	106.299	29.484	7.847	0.998	49.702	18.627	23.9	3.51	0.546	34.679	22.901	15.023	4.259	N/A
0.2	1617	43.524	11.248	9.89	4.322	0.608	14.009	24.929	13.604	1.201	N/A	18.533	16.224	13.681	0.734	N/A
0.2	1346	58.001	44.757	12.23	0.484	N/A	22.136	28.424	4.742	N/A	N/A	15.039	10.436	0.718	N/A	N/A
0.4	2401	34.226	19.937	16.583	3.448	4.244	59.389	27.737	28.579	8.409	1.45	66.487	65.193	36.551	14.883	1.576
0.4	2099	15.008	16.193	18.268	8.034	1.7	29.375	16.614	10.998	4.852	0.952	10.92	15.647	11.076	2.59	N/A
0.4	1881	14.227	32.885	14.944	3.619	0.624	27.861	9.844	8.736	0.874	N/A	10.28	13.604	10.81	N/A	N/A
0.4	1597	8.689	7.348	7.395	0.437	N/A	10.265	9.329	1.233	N/A	N/A	5.132	2.511	N/A	N/A	N/A
0.6	2557	27.768	18.159	29.407	7.145	1.95	48.906	49.795	33.993	17.02	1.435	184.253	125.566	32.339	8.845	1.42
0.6	2336	25.725	17.129	11.841	7.098	1.482	23.307	29.391	13.026	5.366	0.874	20.811	31.371	26.942	2.652	N/A
0.6	2184	10.28	7.566	9.89	2.371	0.624	7.114	5.538	6.677	1.294	N/A	8.674	6.443	4.243	N/A	N/A
0.6	2002	2.434	1.763	2.792	N/A	N/A	1.092	0.967	1.092	N/A	N/A	0.858	1.092	N/A	N/A	N/A
0.8	2713	46.598	2.153	2.012	2.901	1.217	53.586	36.458	24.975	8.955	1.186	20.905	27.987	18.533	11.654	N/A
0.8	2552	12.121	14.493	10.468	2.605	1.154	27.222	17.019	13.151	1.763	N/A	15.35	14.461	7.831	N/A	N/A
0.8	2457	6.63	7.379	9.688	2.012	0.592	1.95	2.168	4.618	N/A	N/A	1.264	2.308	3.136	N/A	N/A
0.8	2307	0.562	1.326	1.014	N/A	N/A	0.359	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
1	2826	9.641	10.935	2.589	1.607	0.655	51.885	44.195	20.108	5.772	0.842	8.236	7.768	3.759	4.961	0.624
1	2762	11.014	4.789	4.539	2.745	0.608	51.684	40.67	22.932	3.525	N/A	10.031	11.217	4.93	2.168	N/A
1	2726	3.822	2.871	1.56	1.279	0.624	2.839	2.308	1.28	0.686	N/A	12.09	8.252	5.913	0.905	N/A
1	2725	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A

manner, we analyzed the effect of selecting various values of D_{Man} on the fitness value for a fixed choice of (α, β). Although, in some cases, no change is observed in the fitness value by increasing the value of D_{Man}, as expected, the trend of change is always non-decreasing and usually tends to be increasing. It should be noted that, generally, there are usually apparent discrepancies for cases with different values of Q. Results show that an increase in the value of Q is associated with an upward move in the value of cost.

Our experiments clearly show that increasing the value of Q and D_{Man} generally leads to solutions with more establishment costs. The role of the proposed model is to generate alternatives to be analyzed and to show how solutions may change using different parameter values. To put it another way, the model of this paper could be used as the model-base of a Decision Support System (DSS). As an example, take the analysis depicted in Figure 6, which gives optimal values of the problem for

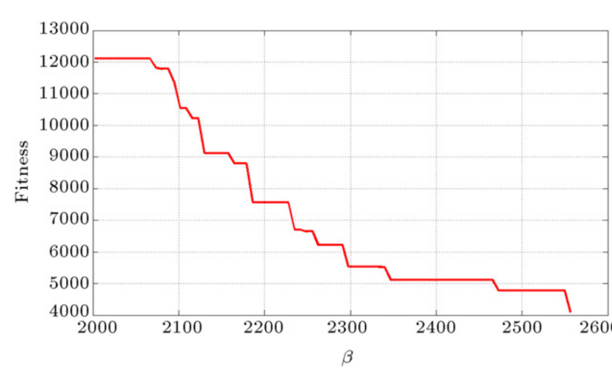

Figure 6: Depicting the optimal values for different values of β.

different values of β. This can provide decision maker(s) with a useful knowledge of model behavior. Obviously, the decision

Table 6: Result of the ANOVA test on the runtimes.

Source	Sum Sq.	d.f.	Mean Sq.	F	Prob $> F$
α	9 456.62538	4	2364.156	10.64596	**1.89E−07**
β	17 987.7097	2	8993.855	40.49996	**2.91E−14**
D_{Man}	17 345.957	3	5781.986	26.03669	**3.94E−13**
Q	1 054.17285	2	527.0864	2.373507	0.097373
$\alpha^*\beta$	5 083.86839	8	635.4835	2.861627	**0.005918**
α^*D_{Man}	2 696.57387	12	224.7145	1.011905	0.442174
α^*Q	4 978.73937	8	622.3424	2.802452	**0.006895**
β^*D_{Man}	5 479.88554	6	913.3143	4.112718	**0.000845**
β^*Q	5 359.11248	4	1339.778	6.033115	**0.000181**
$D^*_{\mathrm{Man}}Q$	693.582174	6	115.597	0.520542	0.791857
Error	27 536.7681	124	222.0707		
Total	97 672.9948	179			

is made using these kinds of analysis, management policy, and available budget, etc.

In order to examine the sensitivity of problem runtime to its parameters, a standard analysis of variance (ANOVA) was carried out, and results are shown in Table 6. At a 99% level of confidence, the main conclusion to draw from this table is that parameters α, β, D_{Man} all affect runtimes to a great extent. Results obtained in Tables 2–4 clearly show that, generally, higher values of parameter β lead to more time needed to solve the problem. Moreover, most times, higher values of runtimes are associated with lower values of α and D_{Man}. Take note that results do not show any specific relation between runtimes and the value of Q. These results can be extremely important in the design of a simulation algorithm, such as Davari et al. [73], where one or some parameters of the problem are fuzzy.

5. Conclusion and future research areas

In the present paper, the hub backup covering problem with mandatory dispersion among hubs is set out under the multiple allocation scheme. We presented a mathematical formulation for the problem, with constraints of backup coverage and mandatory dispersion. Computational experiments conducted using the well-known CAB dataset have given useful insights into this problem. Results showed that, on average, for the CAB dataset and using the specified computer specifications, running the problem needs 15 s, while the maximum needed time is 185. The results soundly confirm that changes in the value of Q can drastically affect the fitness of solutions. Besides, for a given set of parameters, the value of D_{Man} plays a pivotal role in the scatter of hub nodes in the solution space. Hence, the definition of proper values for D_{Man} and Q is of paramount importance for decision makers to design an efficient hub-and-spoke network.

In a nutshell, the point of this paper is to propose a new type of HLP, which seems more practical in real-world problems. This paper contributes to the literature of hub location problem, as it presents a new HLP with backup coverage and mandatory dispersion of hubs. Moreover, the paper evaluates the sensitivity of the problem to its parameters and gives useful insights to interested readers.

Our paper can be extended in various aspects. A recent stream of research in the literature incorporates using various heuristics in order to solve combinatorial optimization problems. Since real-world problems could be much bigger than that considered in this paper, heuristics and metaheuristics deserve to be considered for the problem of this paper. Another interesting future research possibility is inclusion of more than one objective in the problem. Moreover, decision makers are now aware of the fact that in the real world, most decisions are made in an imprecise manner, with imprecise goals and constraints [80]. Thus, considering the same problem with uncertainty could be an area for future research. Last, but not least, is the possibility of assuming novel covering problems, such as the variable radius covering (see [81]) problem in HLP.

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